LIBERTY PAPER SET

STD. 12 : Mathematics

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 12**

PART A

1. (B) **2.** (D) **3.** (A) **4.** (B) **5.** (C) **6.** (A) **7.** (A) **8.** (C) **9.** (A) **10.** (B) **11.** (A) **12.** (A) **13.** (A) **14.** (C) **15.** (D) **16.** (B) **17.** (B) **18.** (B) **19.** (A) **20.** (B) **21.** (C) **22.** (A) **23.** (B) **24.** (B) **25.** (D) **26.** (D) **27.** (B) **28.** (A) **29.** (A) **30.** (A) **31.** (A) **32.** (D) **33.** (D) **34.** (A) **35.** (B) **36.** (D) **37.** (A) **38.** (D) **39.** (B) **40.** (C) **41.** (B) **42.** (C) **43.** (A) **44.** (B) **45.** (B) **46.** (A) **47.** (B) **48.** (D) **49.** (A) **50.** (B)

PART B

SECTION A

1.

Here, $x = sec\theta$,

So, $\sqrt{x^2-1} = \sqrt{sec^2\theta - 1} = tan\theta$

Therefore, $cot^{-1} \frac{1}{\sqrt{x^2-1}} = cot^{-1} (cot\ \theta) = \theta = sec^{-1}x$.

is simple form.

2.

$tan^{-1} \frac{1-x}{1+x} = \frac{1}{2}\ tan^{-1} x$

$\therefore\ 2tan^{-1}\left(\frac{1-x}{1+x}\right) = tan^{-1} x$

Suppose, $tan^{-1}\left(\frac{1-x}{1+x}\right) = \alpha$

$\therefore\ \frac{1-x}{1+x} = tan\ x$

$\therefore\ 2\ tan^{-1}\left(\frac{1-x}{1+x}\right) = tan^{-1} x$

$\therefore\ 2x = tan^{-1} x$

$\therefore\ tan\ 2x = x$

$\therefore\ \frac{2\ tan\ \alpha}{1-tan^2\alpha} = x$

$\therefore\ \frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2} = x$

$\therefore\ \frac{2\left(\frac{1-x}{1+x}\right)\cdot(1+x)^2}{(1+x)^2-(1-x)^2} = x$

$\therefore\ \frac{2(1-x)(1+x)}{1+2x+x^2-1+2x-x^2} = x$

$\therefore\ \frac{2-2x^2}{4x} = x$

$1 - x^2 = 2x^2$

$\therefore\ 3x^2 = 1$

$\therefore\ x^2 = \frac{1}{3}$

$\therefore\ x = \pm\frac{1}{\sqrt{3}}$

$\therefore\ x = \frac{1}{\sqrt{3}}$ $(\because\ x > 0)$

Verification :

L.H.S. $= tan^{-1}\left(\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}\right)$

$= tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$

$= tan^{-1}\left(\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}\right)$

$= tan^{-1}\left(\frac{3-2\sqrt{3}+1}{3-1}\right)$

$= tan^{-1}\ (2 - \sqrt{3}\ )$

$= tan^{-1}\left(tan\frac{\pi}{12}\right) = \frac{\pi}{12}$

$$\text{R.H.S.} = \frac{1}{2}\ tan^{-1}\ x$$

$$= \frac{1}{2}\ tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{2}\ tan^{-1}\left(tan\frac{\pi}{6}\right)$$

$$= \frac{1}{2}\cdot\frac{\pi}{6} = \frac{\pi}{12}$$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ **Solution :** $\left\{\frac{1}{\sqrt{3}}\right\}$

3.

➪ Total differentiation of x in both side,

$$\therefore\ x\cdot\frac{dy}{dx} + y + 2y\cdot\frac{dy}{dx} = sec^2x + \frac{dy}{dx}$$

$$\therefore\ x\cdot\frac{dy}{dx} + 2y\cdot\frac{dy}{dx} - \frac{dy}{dx} = sec^2x - y$$

$$\therefore\ \frac{dy}{dx}(x + 2y - 1) = sec^2x - y$$

$$\therefore\ \frac{dy}{dx} = \frac{sec^2x - y}{x + 2y - 1}$$

4.

➪ We observe that sin^2x is an even function. Therefore, this proves the property (8)(i),

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} sin^2\ x\ dx = 2\int_0^{\frac{\pi}{4}} sin^2\ x\ dx$$

$$= 2\int_0^{\frac{\pi}{4}}\left(\frac{1 - cos\ 2x}{2}\right)dx$$

$$= \int_0^{\frac{\pi}{4}}(1 - cos\ 2x)\ dx$$

$$= \left[x - \frac{1}{2}sin\ 2x\right]_0^{\frac{\pi}{4}}$$

$$= \left[\frac{\pi}{4} - \frac{1}{2}\ sin\ \frac{\pi}{2}\right] - 0$$

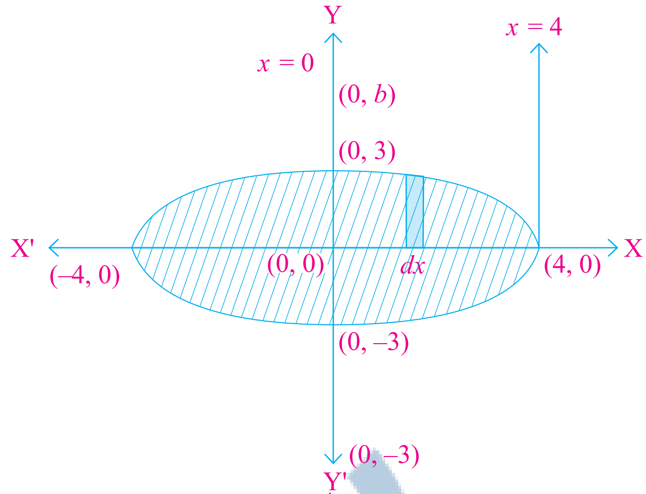
$$= \frac{\pi}{4} - \frac{1}{2}$$

5.

➪ $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$a^2 = 16,\ a = 4\ (a > b)$

$b^2 = 9,\ b = 3$



Required Area :

A = 4 × Area bounded in the first quadrant

$\therefore$ A = 4|I|

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\therefore\ y^2 = 9\left(1 - \frac{x^2}{16}\right)$

$\therefore\ y^2 = \frac{9}{16}\ (16 - x^2)$

$\therefore\ y^2 = \frac{3}{4}\sqrt{16 - x^2}$

$$I = \int_0^4 y\ dx$$

$$I = \int_0^4 \frac{3}{4}\sqrt{16 - x^2}\ dx$$

$$I = \frac{3}{4}\int_0^4 \sqrt{16 - x^2}\ dx$$

$$I = \frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\ sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$I = \frac{3}{4}\left[\left(\frac{4}{2}(0) + 8\ sin^{-1}(1)\right) - \left(0 + sin^{-1}(0)\right)\right]$$

$$I = \frac{3}{4}\left(8\cdot\frac{\pi}{2}\right)$$

$$I = 3\pi$$

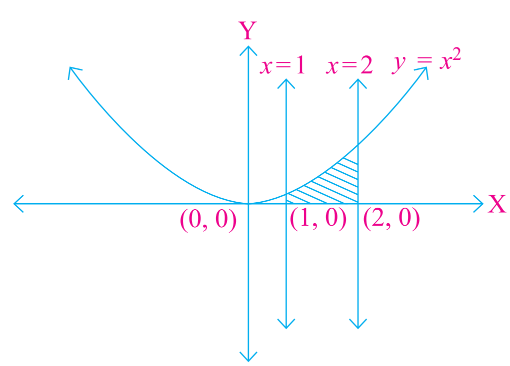
Now, A = 4|I|

= 4|3π|

$\therefore$ A = 12π sq. unit

6.

➪ $x^2 = y$



Required Area,

A = |I|

$\therefore\ I = \int_1^2 y\,dx$

$\therefore\ I = \int_1^2 x^2\,dx$

$\therefore\ I = \left[\frac{x^3}{3}\right]_1^2$

$\therefore\ I = \frac{1}{3}\left((2)^3 - (1)^3\right)$

$\therefore\ I = \frac{7}{3}$

Now, $A = |I| = \left|\frac{7}{3}\right|$

$\therefore\ A = \frac{7}{3}$ sq. unit

7.

$\frac{dy}{dx} = \sin^{-1}x$

$\therefore\ dy = \sin^{-1}x \cdot dx$

→ Take integrate both sides,

$\therefore\ \int 1\,dy = \int \sin^{-1}x\,dx$

$\therefore\ y = [\sin^{-1}x \int 1\,dx] - \int\left[\frac{1}{\sqrt{1-x^2}}\int 1\,dx\right]dx$

$\therefore\ y = x \cdot \sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$\therefore\ y = x\sin^{-1}x + \frac{1}{2}\int (1-x^2)^{\frac{-1}{2}}(-2x)\,dx$

$\therefore\ y = x\sin^{-1}x + \frac{1}{2}\int (1-x^2)^{\frac{-1}{2}} \frac{d}{dx}(1-x^2)dx$

$\therefore\ y = x\sin^{-1}x + \frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c$

$\therefore\ y = x\sin^{-1}x + \sqrt{1-x^2} + c;$

Which is required general solution of given differential equation.

8.

$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$

$\therefore\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = \vec{0}$

$\therefore\ \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6) = \vec{0}$

$\therefore\ 2\mu - 27 = 0;\ \ 2\lambda - 6 = 0;\ 6\mu - 27\lambda = 0$

$\therefore\ \mu = \frac{27}{2};\ \ \lambda = 3$

9.

$\alpha = 90°,\ \beta = 135°,\ \gamma = 45°$

Direction cosins of line,

$\cos\alpha = \cos 90° = 0;$

$\cos\beta = \cos 135° = \cos(90 + 45)°$

$= -\sin 45°$

$= \frac{-1}{\sqrt{2}}$

and $\cos\gamma = \cos 45° = \frac{1}{\sqrt{2}}$

$\therefore$ Direction cosine of line $= 0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

10.

$\frac{x}{2} = \frac{y}{2} = \frac{z}{1} \Rightarrow \frac{x-0}{2} = \frac{y-0}{2} = \frac{z-0}{1}$

$\therefore\ L : \vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$

$\therefore\ \vec{b_1} = 2\hat{i} + 2\hat{j} + \hat{k} \Leftarrow a_1 + \lambda b_1$

and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

$\therefore\ M : \vec{r} = (5\hat{i} + 2\hat{j} + 3\hat{k}) + \mu(4\hat{i} + \hat{j} + 8\hat{k})$

$\therefore\ \vec{b_2} = 4\hat{i} + \hat{j} + 8\hat{k}$

If the angle between L and M is α,

$\cos\alpha = \frac{|\vec{b_1}\cdot\vec{b_2}|}{|\vec{b_1}||\vec{b_2}|}$ .... (1)

$\vec{b_1}\cdot\vec{b_2} = (2\hat{i} + 2\hat{j} + \hat{k})\cdot(4\hat{i} + \hat{j} + 8\hat{k})$

$= 8 + 2 + 8$

$= 18$

Now, $|\vec{b_1}| = \sqrt{4+4+1}$

$= 3$

$|\vec{b_2}| = \sqrt{16+1+64}$

$= 9$

From equation (1),

$\therefore\ \cos\alpha = \frac{|18|}{(3)(9)}$

$\therefore\ \alpha = \cos^{-1}\left(\frac{2}{3}\right)$

Therefore the angle between two lines is $\cos^{-1}\left(\frac{2}{3}\right)$.

11.

(i) A is a subset of B

$A \subset B$

$\therefore\ P(A \cap B) = P(A)$

$\therefore\ P(B \mid A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{P(A)}{P(A)}$

$= 1$ $(\because P(A) \neq 0)$

12.

S = {(g, g), (b, g), (g, b), (b, b)} $n = 4$

(i) Event A : At least one child is male.

A = {(b, g), (g, b), (b, b)}

$r = 3$

$\therefore$ P(A) = $\frac{3}{4}$

Event B : Both are males.

B = {(b, b)}

$r = 1$

$\therefore$ P(B) = $\frac{1}{4}$

A ∩ B = {(b, b)}

$r = 1$

$\therefore$ P(A ∩ B) = $\frac{1}{4}$

$$\therefore P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3}$$

(ii) Event A : Elder child is a female.

A = {(g, b), (g, g)}

$r = 2$

$\therefore$ P(A) = $\frac{2}{4}$

Event B : both are female.

B = {(g, g)}

$r = 1$

$\therefore$ P(B) = $\frac{1}{4}$

P(A ∩ B) = $\frac{1}{4}$

$$\therefore P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1}{2}$$

SECTION B

13.

Suppose $f(x_1) = f(x_2)$.

Note that if x_1 is odd and x_2 is even, then we will have $x_1 + 1 = x_2 - 1$, i.e., $x_2 - x_1 = 2$ which is impossible. Similarly, the possibility of x_1 being even and x_2 being odd can also be ruled out, using the similar argument. Therefore, both x_1 and x_2 must be either odd or even. Suppose both x_1 and x_2 are odd.

Then $f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$.

Similarly, if both x_1 and x_2 are even, then also

$f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$.

Thus, f is one-one.

Also, any odd number $2r + 1$ in the co-domain **N** is the image of $2r + 2$ in the domain **N** and any even number $2r$ in the co-domain **N** is the image of $2r - 1$ in the domain **N**. Thus, f is onto.

14.

$$[1 \quad 2 \quad 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\therefore [1+4+1 \quad 2+0+0 \quad 0+2+2]\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\therefore [6 \quad 2 \quad 4]\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$\therefore$ $[0 + 4 + 4x] = [0]$

$\therefore$ $4 + 4x = 0$

$\therefore$ $4x = -4$

$\therefore$ $x = -1$

15.

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix}$$

$$AB = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix}$$

$= (67)(61) - (87)(47)$

$= 4087 - 4089$

$= -2 \neq 0$

$\therefore$ $(AB)^{-1}$ exists.

$$\text{Thus, } adj\,(AB) = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|}\, adj\, AB$$

$$= \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \quad \text{... (1)}$$

→ For finding A^{-1},

$$|A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix}$$

$$= 15 - 14$$

$$= 1 \neq 0$$

A^{-1} exists.

$$adj\ A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ adj\ A$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix}$$

$$= 54 - 56$$

$$= -2 \neq 0$$

B^{-1} exists.

$$adj\ B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|}\ adj\ B$$

$$= \frac{-1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{-1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{-1}{2}\begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$$

$$= \frac{-1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \quad \dots (2)$$

From equation (1) and (2), $(AB)^{-1} = B^{-1}A^{-1}$

16.

Suppose, $u = x^{sinx}$ and $v = (sinx)^{cosx}$

$\therefore\ y = u + v$

Now, take differentiation of x in both side,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots\dots (1)$$

Here, $u = x^{sinx}$

Now, take log both sides,

$log\ u = sinx \cdot logx$

Now, take differentiation of x both sides,

$$\frac{du}{dx}\frac{1}{u} = sin\ x\ \frac{d}{dx}\ log\ x + logx\ \frac{d}{dx}\ sinx$$

$$= sinx\left(\frac{1}{x}\right) + logx \cdot cosx$$

$$\therefore \frac{du}{dx} = u\left[\frac{sinx}{x} + logx \cdot cosx\right]$$

$$\therefore \frac{du}{dx} = x^{sinx}\left[\frac{sinx}{x} + logx \cdot cosx\right] \quad \dots\dots (2)$$

Now, $v = (sinx)^{cosx}$

Take log both sides,

$log\ v = cosx\ log(sinx)$

Now, take differentiation on both sides,

$$\frac{1}{v}\frac{dv}{dx} = cosx \cdot \frac{d}{dx}\ log\ sinx + logsinx\ \frac{d}{dx}\ cosx$$

$$= cosx \cdot \frac{1}{sinx}\frac{d}{dx}\ sinx + logsinx\ (-\ sinx)$$

$$\therefore \frac{1}{v}\frac{dv}{dx} = cotx \cdot cosx - sinx \cdot logsinx$$

$$\therefore \frac{dv}{dx} = v\ [cotx \cdot cosx - sinx \cdot logsinx]$$

$$\therefore \frac{dv}{dx} = (sinx)^{cosx}\ [cotx \cdot cosx - sinx \cdot logsinx] \quad \dots\dots (3)$$

Put the equation (2) and (3) in equation (1),

$$\frac{dy}{dx} = x^{sinx}\left[\frac{sinx}{x} + logx \cdot cosx\right] + (sinx)^{cosx}\left[cotxcosx - sinx\ log\ sinx\right]$$

17.

For each value of x, the helicopter's position is at point $(x, x^2 + 7)$.

Therefore, the distance between the helicopter and the soldier placed at (3, 7) is

$\sqrt{(x-3)^2 + (x^2+7-7)^2}$, i.e., $\sqrt{(x-3)^2 + x^4}$.

Let $f(x) = (x - 3)^2 + x^4$

or $f'(x) = 2(x - 3) + 4x^3$

$= 2(x - 1)(2x^2 + 2x + 3)$

Thus, $f'(x) = 0$ gives $x = 1$ or $2x^2 + 2x + 3 = 0$ for which there are no real roots.

Also, there are no end points of the interval to be added to the set for which f' is zero, i.e., there is only one point, namely, $x = 1$. The value of f at this point is given by $f(1) = (1 - 3)^2 + (1)^4 = 5$.

Thus, the distance between the solider and the helicopter is $\sqrt{f(1)} = \sqrt{5}$.

Note that $\sqrt{5}$ is either a maximum value or a minimum value.

Since, $\sqrt{f(0)} = \sqrt{(0-3)^2 + (0)^4} = 3 > \sqrt{5}$,

it follows that $\sqrt{5}$ is the minimum value of $\sqrt{f(x)}$.

Hence, $\sqrt{5}$ is the minimum distance between the soldier and the helicopter.

18.

(i) The position vector of the point R dividing the joint of P and Q internally in the ratio 2 : 1 is

$$\overrightarrow{OR} = \frac{2(\vec{a}+\vec{b})+(3\vec{a}-2\vec{b})}{2+1}$$

$$= \frac{5\vec{a}}{3}$$

(ii) The position vector of the point R dividing the joint of P and Q externally in the ratio 2 : 1 is

$$\overrightarrow{OR} = \frac{2(\vec{a}+\vec{b})-(3\vec{a}-2\vec{b})}{2-1}$$

$$= 4\vec{b}-\vec{a}$$

19.

Comparing (1) and (2) with $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ respectively

We get,

$\vec{a_1} = \hat{i} + \hat{j}$, $\vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

and $\vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$.

For, $\vec{a_2} - \vec{a_1} = \hat{i} - \hat{k}$

and $\vec{b_1} \times \vec{b_2} = (2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} - 5\hat{j} + 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$= 3\hat{i} - \hat{j} + 7\hat{k}$$

So, $|\vec{b_1} \times \vec{b_2}| = \sqrt{9+1+49}$

$$= \sqrt{59}$$

Hence, the shortest distance between the given lines is given by,

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$= \left| \frac{3-0+7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}} \text{ unit}$$

20.

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \geq 200$

$x \geq 0$

$y \geq 0$

Option function Z = $5x + 10y$

$x + 2y = 100$ ... (i)

x	0	100
y	50	0

(0, 50) ✓
(100, 0) ×

$2x - y = 0$ ... (ii)

x	0	1
y	0	2

(0, 0) ×
(1, 2)

$2x + y = 200$ ... (iii)

x	0	100
y	200	0

(0, 200) ✓
(100, 0) ×

Solve equation (i) and (ii),

$\therefore x + 2(2x) = 100$

$\therefore 5x = 100$ ...(1)

$\therefore x = 20$ $\therefore y = 40$

$\therefore$ (20, 40) ✓

Solve equation (ii) and (iii),

$$\begin{aligned} 2x + y &= 200 \\ 2x - 2y &= 0 \\ \hline 4x &= 200 \end{aligned}$$

$\therefore x = 50$ $\therefore y = 100$

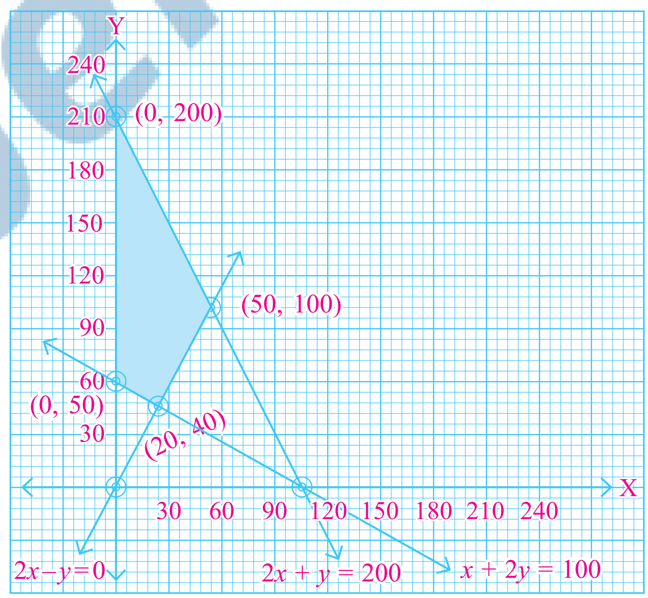
$\therefore$ (50, 100) ✓

Solve equation (i) and (iii),

$$\begin{aligned} x + 2y &= 100 \\ 4x + 2y &= 400 \\ - \quad - \quad &\quad - \\ \hline 3x &= 300 \end{aligned}$$

$\therefore x = 100$ $\therefore y = 0$

$\therefore$ (100, 0) ×



The shaded region in fig. is feasible region determined by the system of constrints which is bounded. The co-ordinates of corner points are (0, 50), (20, 40), (50, 100) and (0, 200).

Corner Point	Corresponding value of $Z = x + 2y$
(20, 40)	100 ← Minimum
(50, 100)	250
(0, 200)	400 ← Maximum
(0, 50)	100 ← Minimum

Thus, the Maximum value of Z is 500 and Minimum value of Z is 100.

21.

Event E_1 : Number 5 or 6 obtained on dice.

Event E_2 : Number 1, 2, 3 or 4 is obtained on dice.

$P(E_1) = \frac{2}{6} = \frac{1}{3}$

$P(E_2) = \frac{4}{6} = \frac{2}{3}$

Event A : only one head is obtained.

Probability that if exactly one head is obtained if number 1, 2, 3 or 4 obtained on dice,

$P(A \mid E_1)$ = Probability that number 5 or 6 is obtained toss the coin thrice

$\therefore \; P(A \mid E_1) = \frac{3}{8}$

$P(A \mid E_2)$ = Probability that number 1, 2, 3, 4 is obtained on dice then coin

$\therefore \; P(A \mid E_2) = \frac{1}{2}$

$$\therefore \; P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2)$$

$$= \frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}$$

$$= \frac{1}{8} + \frac{1}{3}$$

$$= \frac{11}{24}$$

$$\therefore \; P(E_2 \mid A) = \frac{P(A \mid E_2) \cdot P(E_2)}{P(A)}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{11}{24}}$$

$$= \frac{8}{11}$$

SECTION C

22.

Here, $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$$A^T = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A^T)$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & 3-2 & -1-4 \\ -2+3 & -2-2 & 1-5 \\ -4-1 & -5+1 & 2+2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

$\therefore \; P = P^T$

$\therefore$ P is symmetric matrix.

$$Q = \frac{1}{2}(A - A^T)$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & 3+2 & -1+4 \\ -2-3 & -2+2 & 1+5 \\ -4+1 & -5-1 & 2-2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

$$Q^T = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

$\therefore \; Q = -Q^T$

$\therefore$ Q is skew symmetric matrix.

$$P + Q = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$= A$$

23.

Suppose, the cost of 1 kg onion = ₹ x

the cost of 1 kg weat = ₹ y

the cost of 1 kg rice = ₹ z

The cost of 4 kg onion, 3 kg wheat, 2 kg rice is ₹ 60

$\therefore \; 4x + 3y + 2z = 60$

The cost of 2 kg onion, 4 kg wheat, 6 kg rice is ₹ 90

$\therefore \; 2x + 4y + 6z = 90$

The cost of 6 kg onion, 2 kg wheat, 3 kg rice is ₹ 70

$\therefore \quad 6x + 2y + 3z = 70$

$4x + 3y + 2z = 60$

$2x + 4y + 6z = 90$

$6x + 2y + 3z = 70$

The matrix form can be represented as,

$$\therefore \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$\therefore$ AX = B

$$\text{Where, } A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

AX = B

$\therefore$ X = A^{-1}B

For finding A^{-1},

$$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix}$$

$= 4(12 - 12) - 3(6 - 36) + 2(4 - 24)$

$= 0 - 3(-30) + 2(-20)$

$= 90 - 40$

$= 50 \neq 0$

$\therefore$ A^{-1} exists.

For finding *adj* A,

Co-factor element of 4 $\quad A_{11} = (-1)^2 \begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix}$

$= 1(12 - 12)$

$= 0$

Co-factor element of 3 $\quad A_{12} = (-1)^3 \begin{vmatrix} 2 & 6 \\ 6 & 3 \end{vmatrix}$

$= (-1)(6 - 36)$

$= 30$

Co-factor element of 2 $\quad A_{13} = (-1)^4 \begin{vmatrix} 2 & 4 \\ 6 & 2 \end{vmatrix}$

$= 1(4 - 24)$

$= -20$

Co-factor element of 2 $\quad A_{21} = (-1)^3 \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}$

$= (-1)(9 - 4)$

$= -5$

Co-factor element of 4 $\quad A_{22} = (-1)^4 \begin{vmatrix} 4 & 2 \\ 6 & 3 \end{vmatrix}$

$= 1(12 - 12)$

$= 0$

Co-factor element of 6 $\quad A_{23} = (-1)^5 \begin{vmatrix} 4 & 3 \\ 6 & 2 \end{vmatrix}$

$= (-1)(8 - 18)$

$= 10$

Co-factor element of 6 $\quad A_{31} = (-1)^4 \begin{vmatrix} 3 & 2 \\ 4 & 6 \end{vmatrix}$

$= 1(18 - 8)$

$= 10$

Co-factor element of 2 $\quad A_{32} = (-1)^5 \begin{vmatrix} 4 & 2 \\ 2 & 6 \end{vmatrix}$

$= (-1)(24 - 4)$

$= -20$

Co-factor element of 3 $\quad A_{33} = (-1)^6 \begin{vmatrix} 4 & 3 \\ 2 & 4 \end{vmatrix}$

$= 1(16 - 6)$

$= 10$

$$adj\ A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ adj\ A$$

$$= \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

X = A^{-1}B

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore \quad x = 5,\ y = 8,\ z = 8$

Therefore, the cost of 1 kg onion = ₹ 5

the cost of 1 kg wheat = ₹ 8

the cost of 1 kg rice = ₹ 8.

24.

Given, $y = 3e^{2x} + 2e^{3x}$

So, $\dfrac{dy}{dx} = 6e^{2x} + 6e^{3x} = 6(e^{2x} + e^{3x})$

$\therefore \dfrac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x} = 6(2e^{2x} + 3e^{3x})$

So, $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y$

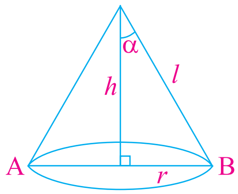
$= 6(2e^{2x} + 3e^{3x}) - 30(e^{2x} + e^{3x}) + 6(3e^{2x} + 2e^{3x})$

$= 0$

25.

Suppose, radius of cone is r, height is h and slant height is l.

$\therefore\ l^2 = h^2 + r^2$



Suppose, semi vertical angle is α.

$$\therefore\ \sin\alpha = \frac{r}{l}$$

$$\therefore\ \alpha = \sin^{-1}\left(\frac{r}{l}\right)$$

Surface area of cone (S) = $\pi rl + \pi r^2$ .... (1)

$$\therefore\ S = \pi r\left(\sqrt{h^2+r^2}\right) + \pi r^2$$

$$\therefore\ S = \pi r\left(\sqrt{h^2+r^2}\right) + r)$$

$$\therefore\ \frac{S}{\pi r} = \sqrt{h^2+r^2} + r$$

$$\therefore\ \frac{S}{\pi r} - r = \sqrt{h^2+r^2}$$

$$\therefore\ \left(\frac{S}{\pi r} - r\right)^2 = h^2 + r^2 \qquad \text{........... (2)}$$

Volume of cone (V) = $\frac{1}{3}\pi r^2 h$

$$\therefore\ V^2 = \frac{1}{9}\pi^2 r^4 h^2$$

$$\therefore\ V^2 = \frac{1}{9}\pi^2 r^4 \left(\left(\frac{S}{\pi r} - r\right)^2 - r^2\right)$$

($\because$ From equation (2))

$$\therefore\ V^2 = \frac{1}{9}\pi^2 r^4 \left(\frac{S^2}{\pi^2 r^2} - \frac{2S}{\pi} + r^2 - r^2\right)$$

$$\therefore\ V^2 = \frac{1}{9}\pi^2 r^4 \left(\frac{S^2 - 2S\pi r^2}{\pi^2 r^2}\right)$$

$$\therefore\ V^2 = \frac{1}{9} r^2 (S^2 - 2S\pi r^2)$$

$$\therefore\ V^2 = \frac{r^2 S^2}{9} - \frac{2S\pi r^4}{9}$$

$$f(r) = \frac{r^2 S^2}{9} - \frac{2S\pi r^4}{9}$$

$$\therefore\ f'(r) = \frac{2r\cdot S^2}{9} - \frac{8r^3 S\pi}{9}$$

$$\therefore\ f''(r) = \frac{2S^2}{9} - \frac{24r^2 S\pi}{9} \qquad \text{............ (3)}$$

→ For finding maximum volume of cone,

$$f'(r) = 0$$

$$\therefore\ \frac{2rS^2}{9} - \frac{8r^3 S\pi}{9} = 0$$

$$\therefore\ \frac{2r S^2}{9} = \frac{8S\pi r^3}{9}$$

$$\therefore\ S = 4\pi r^2$$

→ Put S = $4\pi r^2$ in equation (3),

$$\therefore\ f''(r) = \frac{2}{9}(16\pi^2 r^4) - \frac{24r^2\pi(4\pi r^2)}{9}$$

$$= \frac{32\pi^2 r^4}{9} - \frac{96\pi^2 r^4}{9}$$

$$\therefore\ f''(r) = \frac{-64\pi^2 r^4}{9} < 0 \qquad (\because r^4 > 0)$$

$\therefore$ f has maximum value.

→ Put S = $4\pi r^2$ in equation (1),

$$\therefore\ \pi rl + \pi r^2 = 4\pi r^2$$

$$\therefore\ \pi rl = 3\pi r^2$$

$$\therefore\ l = 3r$$

$$\therefore\ \frac{l}{r} = 3$$

$$\therefore\ \frac{r}{l} = \frac{1}{3}$$

→ Semi-vertical angle $= \sin^{-1}\left(\frac{r}{l}\right)$

$$= \sin^{-1}\left(\frac{1}{3}\right)$$

26.

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x)}{\sqrt{1-(1-\sin 2x)}}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x)\,dx}{\sqrt{1-(\sin^2 x + \cos^2 x - 2\sin x\cos x)}}$$

$$= \int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x)\,dx}{\sqrt{1-(\sin x - \cos x)^2}}$$

Now, take $\sin x - \cos x = t$,

$\therefore$ $(\cos x + \sin x)\,dx = dt$

When, $x = \frac{\pi}{6}$ then $t = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$

When, $x = \frac{\pi}{3}$ then $t = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$

$$I = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$$

$$I = \left[\sin^{-1}(t)\right]_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}$$

$$= \sin^{-1}\left[\frac{\sqrt{3}-1}{2}\right] - \sin^{-1}\left[\frac{1-\sqrt{3}}{2}\right]$$

$$= \sin^{-1}\left[\frac{\sqrt{3}-1}{2}\right] + \sin^{-1}\left[\frac{\sqrt{3}-1}{2}\right]$$

$$I = 2\sin^{-1}\left[\frac{\sqrt{3}-1}{2}\right]$$

27.

Method 1 :

The given function is

$y = e^{ax}[c_1 \cos bx + c_2 \sin bx]$ ... (1)

Differentiating both sides of equation (1) with respect to x

We get,

$$\frac{dy}{dx} = e^{ax}[-bc_1 \sin bx + bc_2 \cos bx] + [c_1 \cos bx + c_2 \sin bx]\, e^{ax} \cdot a$$

$$\therefore \frac{dy}{dx} = e^{ax}[(bc_2 + ac_1)\cos bx + (ac_2 - bc_1)\sin bx] \quad ...\ (2)$$

→ Differentiating both sides of equation (2) with respect to x

We get,

$$\frac{d^2y}{dx^2} = e^{ax}[(bc_2 + ac_1)(-b \sin bx) + (ac_2 - bc_1)(b \cos bx)] + [(bc_2 + ac_1)\cos bx + (ac_2 - bc_1)\sin bx]\, e^{ax} \cdot a$$

$$= e^{ax}[(a^2c_2 - 2abc_1 - b^2c_2)\sin bx + (a^2c_1 + 2abc_2 - b^2c_1)\cos bx]$$

→ Substituting the value of $\frac{d^2y}{dx^2}, \frac{dy}{dx}$ and y in the given differential equation.

We get,

$$\text{L.H.S.} = e^{ax}[(a^2c_2 - 2abc_1 - b^2c_2)\sin bx + (a^2c_1 + 2abc_2 - b^2c_1)\cos bx] - 2ae^{ax}[(bc_2 + ac_1)\cos bx + (ac_2 - bc_1)\sin bx] + (a^2 + b^2)e^{ax}[c_1 \cos bx + c_2 \sin bx]$$

$$= e^{ax}(a^2c_2 - 2abc_1 - b^2c_2 - 2a^2c_2 + 2abc_1 + a^2c_2 + b^2c_2)\sin bx + (a^2c_1 + 2abc_2 - b^2c_1 - 2abc_2 - 2a^2c_1 + a^2c_1 + b^2c_1)\cos bx$$

$$= e^{ax}[0 \times \sin bx + 0 \times \cos bx]$$

$$= e^{ax} \times 0$$

$$= 0$$

= R.H.S.

Hence, the given function is a solution of the given differential equation.

Method 2 :

$$ye^{-ax} = c_1 \cos bx + c_2 \sin bx$$

$$\therefore e^{-ax}y_1 - aye^{-ax} = -bc_1 \sin bx + bc_2 \cos bx$$

$$\therefore y_2e^{-ax} - 2ae^{-ax}y_1 + a^2ye^{-ax} = -b^2c_1 \cos bx - b^2c_2 \sin bx = -b^2\, ye^{-ax}$$

$$\therefore y_2 - 2ay_1 + (a^2 + b^2)\, y = 0$$